$U(1)$ Chiral Gauge Theory with Domain Wall Fermions and Gauge Fixing

S. Basak[a] and Asit K. De[a*]

[a]Saha Institute of Nuclear Physics, 1/AF Salt Lake, Calcutta 700064, India

We investigate a U(1) lattice chiral gauge theory ($L\chi GT$) with the waveguide formulation of the domain wall fermions and with compact gauge fixing. In the reduced model limit, there seems to be no mirror chiral modes at the waveguide boundary.

1. Introduction

Gauge fixing is not necessary in Wilson lattice gauge theory. For gauge-non-invariant theories, if gauge fixing is not done with a target gauge-invariant theory in mind, however, there are nontrivial consequences, namely, the longitudinal gauge degrees of freedom (*dof*) couple to physical *dof*. The well-known example is the Smit-Swift proposal of $L\chi GT$. The obvious remedy is to gauge fix. The Roma proposal involving gauge fixing passed perturbative tests but does not address the problem of gauge fixing of compact gauge fields and the associated problem of lattice artifact Gribov copies. The formal problem is that for compact gauge-fixing a BRST-invariant partition function as well as (unnormalized) expectation values of BRST invariant operators vanish as a consequence of lattice Gribov copies. Shamir and Golterman [1] has proposed to keep the gauge-fixing part of the action BRST non-invariant and tune counterterms to recover BRST in the continuum. In their formalism, the continuum limit is to be taken from within the broken ferromagnetic (FM) phase approaching another broken phase which is called ferromagnetic directional (FMD) phase, with the mass of the gauge field vanishing at the FM-FMD transition. This was tried out in a U(1) Smit-Swift model [3] and so far all indications are that in the pure gauge sector, QED is recovered in the continuum limit and in the reduced model limit free chiral fermions in the appropriate chiral representation are obtained.

We want to extend this idea to other previous proposals of a $L\chi GT$ which supposedly failed due to above mentioned coupling of longitudinal gauge *dof* to physical *dof*. For this pupose we have chosen the waveguide formulation of the domain wall fermion where mirror chiral modes appeared at the waveguide boundary in addition to the chiral modes at the domain wall or antiwall to spoil the chiral nature of the theory [2].

2. Gauge-fixed Domain Wall Action

For Kaplan's free domain wall fermions on a $4+1$-dimensional lattice of size $L^4 L_s$ where L_s is the 5th dimension, with periodic boundary conditions in the 5th or s-direction, there is always an anti-domain wall and the model possesses two zero modes of opposite chirality, one bound to the domain wall at $s = 0$ and the other bound to the antiwall at $s = \frac{L_s}{2}$. With the domain wall mass $m(s)$ taken as

$$m(s) = \begin{array}{ll} -m_0 & 0 < s < \frac{L_s}{2} \\ 0 & s = 0, \frac{L_s}{2} \\ m_0 & \frac{L_s}{2} < s < L_s \end{array}$$

the wall mode is lefthanded (LH), the antiwall mode is righthanded (RH). If $m_0 L_s \gg 1$, these modes would have exponentially small overlap. These chiral modes exist for momenta p below a critical momentum p_c, i.e. $|\hat{p}| < p_c$, where $\hat{p}^2 = 2\sum_\mu [1 - cos(p_\mu)]$ and $p_c^2 = 4 - 2m_0/r$.

Since the LH and RH modes of the fermion are separated in s space, one can attempt to couple these two in different ways to a gauge field. A 4-dimensional gauge field which is same for all s-slices can be coupled to fermions only for

---

a restricted number of s-slices around the anti-domain wall [2] with a view to coupling only to the RH mode at the antiwall. The gauge field is thus confined within a *waveguide*, $WG = (s : s_0 < s \leq s_1)$. Our choice of s_0 and s_1 is such that the anti-domain wall is located at the center of the WG, and at the same time far enough from the domain or anti-domain wall that the zeromodes are exponentially small at the WG boundary [2]: $s_0 = \frac{L_s+2}{4} - 1$, $s_1 = \frac{3L_s+2}{4} - 1$. With this choice $L_s - 2$ has to be a multiple of four. The global symmetry of the model and the gauge transformations on the fermion field in and outside WG remain exactly the same as in ref. [2]. Obviously, the hopping terms from s_0 to s_0+1 and that from s_1 to $s_1 + 1$ would break the local gauge invariance of the action. This is taken care of by gauge transforming the action and thereby picking up the pure gauge *dof* or a radially frozen scalar field ϕ (Stückelberg field) at the WG boundary. So the gauge invariant WG action now becomes, retaining only the s index, taking the Wilson $r = 1$ and lattice constant $a = 1$,

$$
\begin{aligned}
S_F = & \sum_{s \in WG} \overline{\psi}^s \left(\slashed{D}(U) - W(U) + m(s) \right) \psi^s \\
& + \sum_{s \notin WG} \overline{\psi}^s \left(\slashed{\partial} - w + m(s) \right) \psi^s \\
& - \sum_{s \neq s_0, s_1} \left(\overline{\psi}^s P_L \psi^{s+1} + \overline{\psi}^{s+1} P_R \psi^s \right) + \sum_s \overline{\psi}^s \psi^s \\
& - y \left(\overline{\psi}^{s_0} \phi^\dagger P_L \psi^{s_0+1} + \overline{\psi}^{s_0+1} \phi P_R \psi^{s_0} \right) \\
& - y \left(\overline{\psi}^{s_1} \phi P_L \psi^{s_1+1} + \overline{\psi}^{s_1+1} \phi^\dagger P_R \psi^{s_1} \right)
\end{aligned}
\tag{1}
$$

where y is the Yukawa coupling at the WG boundaries and the projector P_L (P_R) is $(1-\gamma_5)/2$ $((1+\gamma_5)/2)$. The $\slashed{D}(U)$ and $W(U)$ are the gauge covariant Dirac operator and the Wilson term respectively.

The gauge-fixed pure gauge action for U(1), where the ghosts are free and decoupled, is:

$$
S_B(U) = S_g(U) + S_{gf}(U) + S_{ct}(U)
\tag{2}
$$

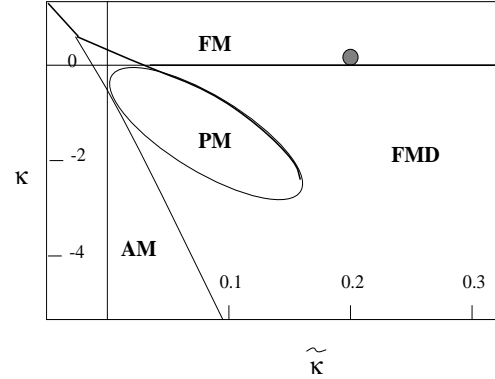where, S_g is the usual Wilson plaquette action, gauge fixing term S_{gf} and the gauge field mass

Figure 1. *Schematic quenched phase diagram*

counter term S_{ct} are given by,

$$
S_{gf}(U) = \tilde{\kappa} \left(\sum_{xyz} \Box(U)_{xy} \Box(U)_{yz} - \sum_x B_x^2 \right), \tag{3}
$$

$$
S_{ct}(U) = -\kappa \sum_{x\mu} \left(U_{\mu x} + U_{\mu x}^\dagger \right), \tag{4}
$$

where $\Box(U)$ is the covariant lattice laplacian and

$$
B_x = \sum_\mu \left(\frac{V_{\mu x - \hat{\mu}} + V_{\mu x}}{2} \right)^2 \tag{5}
$$

with $V_{\mu x} = \frac{1}{2i} \left(U_{\mu x} - U_{\mu x}^\dagger \right)$ and $\tilde{\kappa} = 1/(2\xi g^2)$. S_{gf} has a unique absolute minimum at $U_{\mu x} = 1$, validating weak coupling perturbation theory (WCPT) around $g = 0$ or $\tilde{\kappa} = \infty$ and in the naive continuum limit it reduces to $\frac{1}{2\xi} \int d^4x (\partial_\mu A_\mu)^2$.

Obviously, the action $S_B(U)$ is not gauge invariant. By giving it a gauge transformation the resulting action $S_B(\phi_x^\dagger U_{\mu x} \phi_{x+\hat{\mu}})$ is gauge-invariant with $U_{\mu x} \to g_x U_{\mu x} h_{x+\hat{\mu}}^\dagger$ and $\phi_x \to g_x \phi_x$, $g_x \in U(1)$. By restricting to the trivial orbit, we arrive at the so-called **reduced model** action

$$
S_{reduced} = S_F(U = 1) + S_B(\phi_x^\dagger \ 1 \ \phi_{x+\hat{\mu}}) \tag{6}
$$

where $S_B(\phi_x^\dagger \ 1 \ \phi_{x+\hat{\mu}})$ now is a higher-derivative scalar field theory action.

3. Results in the Reduced Model

In the quenched approximation, we have first numerically confirmed the phase diagram in [3]
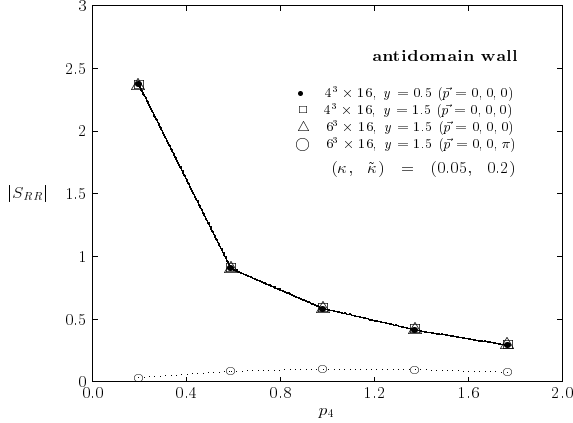
Figure 2. *RR-propagator at antidomain wall* ($L_s = 22$; *a.p.b.c. in* L_4)

Figure 3. *LL-propagator at waveguide boundary* ($L_s = 22$, *a.p.b.c. in* L_4)

of the model in $(\kappa, \tilde{\kappa})$ plane. The phase diagram shown schematically in Fig.1 has the interesting feature that for large enough $\tilde{\kappa}$, there is a continuous phase transition from FM to FMD phase. FMD phase is characterised by loss of rotational invariance and the continuum limit is to be taken from the FM side of the transition. In the full theory with dynamical gauge fields, the gauge symmetry reappears at this transition and the gauge boson mass vanishes, but the longitudinal gauge *dof* remain decoupled. For calculating the fermion propagators, we have chosen the point $\kappa = 0.05$, $\tilde{\kappa} = 0.2$ (gray blob in Fig.1). Numerically on 4^316 and 6^316 lattices with $L_s = 22$ we look for chiral modes at the domain wall ($s = 0$), the antidomain wall ($s = 11$), and at the WG boundaries ($s = 5, 6$ and $s = 16, 17$). Error bars in Figs.2 and 3 are smaller than the symbols. Fig.2 shows the RR-propagator $|S_{RR}|$ at $s = 11$ as a function of a component of momentum p_4 for both $\vec{p} = (0, 0, 0)$ (physical mode) and $(0, 0, \pi)$ (first doubler mode) at different y-couplings. From the figure, it is clear that the doubler does not exist and the physical RR-propagator seems to have a pole at $p = (0, 0, 0, 0)$. The curve stays the same irrespective of y-coupling and lattice size. We have also checked that it coincides exactly with the free RR-propagator (corresponding to $y\phi = 1$) at $s = 11$. Also similar analysis with the LL-propagator at $s = 11$ does not show any pole.
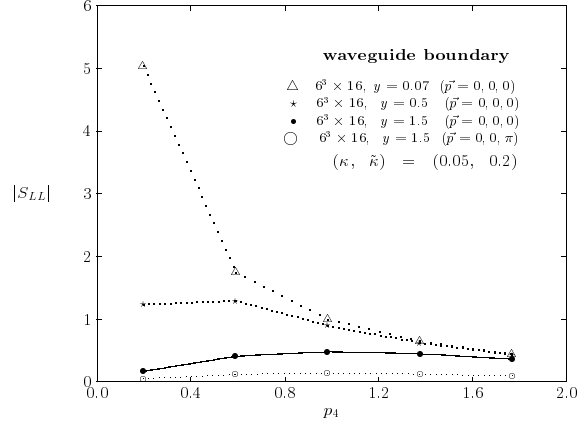
We conclude that there is *only a free RH fermion* at the antidomain wall. From similar data not shown here, we conclude that at the domain wall, there is a *only free LH fermion*. Fig.3 shows the LL propagator at $s = 6$ (waveguide boundary, just inside). Here too doublers do not exist. While there is a hint of a pole for $y = 0.07$, the data for $y = 0.5$, 1.5 does not favor any pole at zero momentum. Taking all our numerical data into account for the boundary propagators, there does not seem to exist any chiral modes there for $y \sim 1$. For very small y the situation is tricky, because strictly at $y = 0$, there are mirror boundary modes present, as can be seen from considerations of fermion current in s-direction and also from numerical simulation. We have done WCPT around $\tilde{\kappa} = \infty$ and $y = 1$ and it supports the numerical data for $y \sim 1$.

In spite of having the longitudinal gauge *dof* explicitly in the action, it seems that in the reduced model we end up only with free undoubled chiral fermions at the domain wall and the antiwall with no mirror modes at the WG boundaries.

REFERENCES

1. M.F.L. Golterman and Y. Shamir, Phys. Lett. B399. (1997) 148.
2. M.F.L. Golterman, K. Jansen, D.N. Petcher and J.C. Vink, Phys. Rev. D49 (1994) 1606.
3. W. Bock et.al., Nucl.Phys.B (Proc.Suppl.) 63(1998)147; Phys.Rev.Lett 80(1998)3444.